NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

'AN EXPLORATORY INPUT-OUTPUT MODEL
FOR SUPPORT COST ESTIMATION

by

Clemente Pionilla Mariano

September 1976

Thesis Advisor: J. A. Larson

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AN EXPLORATORY INPUT-OUTPUT MODEL

FOR SUPPORT COST ESTIMATION

by

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A cost model to estimate support costs in the Philippine Army is developed, Leontief's input-output technique is appraised for applicability in the cost model. A role for input-output cost models is related with cost analysis. In discussing the data aspects of the cost model, the use of proxy variables and their validation are presented. Correlation analysis is suggested as a tool for choosing a valid proxy variable. The matter of fixed costs is discussed also, and its treatment in the cost model is suggested. Using hypothetical data, an example on the use of the model in a Philippine Army setting is presented. Support units, tactical forces, and budgetary programs comprise the major elements of the system in the cost model.

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I. INTRODUCTION

The level and structure of military forces in many countries have continually varied with the changing need for readiness over time. Policy makers in the military establishment require adequate information about the implications of formulated alternatives in the process of selecting the best course of action to pursue. One concern in decision analysis is resource constraints. They exert considerable influence in determining the final choice among the various options which confront the decision-maker. A sufficient knowledge of the resource implications of each alternative course of action under study could be of value to the decision-maker.

Among the essential information in the analysis and evaluation of planning alternatives is cost. The allocation of resources may depend much on the cost side of activities. In planning and decision making, the knowledge about costs of alternatives oftentimes must be derived from estimates. The urgency to evolve logical plans in many situations requires that cost estimates be derived at a short notice.

Information on cost may be predictive in nature because of the effects of related variables. Techniques on cost estimation range from a direct computational approach to more complicated ones requiring a series of different steps. Support cost for example, is one category of cost in the military that may not easily be traceable using conventional procedures of estimation. Such cost in general is a function

of the level and structure of forces. In terms of weapon systems and tactical forces, the nature of support costs is not explicit. Support organizations are frequently interrelated in the sense that they also provide support to one another. The total support output that such an organization should generate is composed of the output that goes to non-support organizations which are mostly tactical forces, and output that goes to support organizations, including itself possibly.

Leontief's input-output analysis is a convenient vehicle for capturing the implicit relationship of support costs. It could functionally relate support costs to changes in force level and structure. This single characteristic alone could be important in analysis. A cost model for estimating support costs based on this concept could promise a quick and fairly accurate estimate desirable to most resources managers.[1]

This paper aims to establish a methodology based on Leontief's input-output analysis suitable to some of the needs of the Philippine Army. As observed by this writer, the need for a fast and reliable cost estimation technique had been felt by the planners of organizational expansion. Considerable difficulty was encountered in developing a plan for a phased buildup of forces to meet internal exigency consistent with resource restrictions during the

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[1] J. Augusta and N. Hibbs, Estimating U.S. Navy Support Costs, Center for Naval Analysis Research Contribution 180, p. i

period 1972-1974. The attainment of a balance between tactical and support organizations was hindered by the complexity of interrelatedness between support organizations on one side, and between support organizations and tactical forces on the other. Successive modifications led to the desired organizational set-up, but at an extreme cost in time and effort. This writer believes that the planners would have encountered lesser difficulty in formulating the alternatives and assessing their implicit consequences if a cost estimation methodology capable of quick and fairly accurate forecasts was at hand.

In developing the methodology for cost estimation, this paper initially reviews the theoretical framework of Leontief's input-output analysis in order to establish the foundation of the technique to be pursued. Cost analysis as typically practiced, is then briefly discussed to show potential areas in military planning and decision making processes where cost estimation procedures could be of use. The cost model is presented using hypothetical data to simulate the application of the proposed technique in a Philippine Army setting.

II. THE CONCEPTUAL FRAMEWORK OF INPUT-OUTPUT ANALYSIS

A. ORIGIN[2]

Most of the written publications dealing with input-output analysis trace its origin to Wassily Leontief of Harvard University, who beginning in the 1930's developed a general theory of production based on economic interdependence. The basic idea of economic interdependence though, was originated by Quesnay when he showed how goods and services circulate among the three socio-economic classes he identified as the land owner, farmer, and manufacturer or trader, in the "Tableau Economique" that he presented. Quesnay showed how income in each class relates to the interdependent activities of the classes as regards consumption and production. It was Walras later who expounded on the concept of general interdependence of economic activities, and introduced the concept of "coefficient of production" to represent the amount of each productive service required to produce a unit of a product. Input-output analysis then had developed into one popular innovation in economic analysis. Just like any other new idea, there had been some controversial issues among economists pertaining to the assumptions in the analysis.

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[2] Hatanaka, Michio, The Workability of Input-Output Analysis, pp. 7-11, Fachverlag Fur Wirtshaftstheoie und Okonometrie Ludwigshafen Am Rhein, 1960

B. THE CONCEPT

One way of presenting the concept of input-output analysis is by way of a table showing the flow of goods from producer to producer, and from producer to final user.[3] In this context, a final user is only a consumer of goods and does not produce any good which may be required in the productive processes of the other industries or producers in the system. The table illustrated in Figure 1 shows the interaction between industries and final user. In this model, an exogenous input called primary factor is introduced. A primary factor is not produced within the system, but may be an essential input to some, if not all of the productive processes. The table shows that Industry 1 has portions of its output going to itself, to Industry 2, and to final consumption. Labor, the primary factor, is input to both industries. If the total output of the i^{th} industry is represented by X_i, the relationship between total output, intermediate use, and final consumption in algebraic notation is as follows:

$$X_i = x_{i1} + x_{i2} + C_i \qquad (1)$$

where x_{ij} is the i^{th} industry's input to the j^{th} industry, and C_i is the final consumption or demand for the i^{th} industry's output. From the table, the sum of x_{i1} and x_{i2} is that part of the i^{th} industry's output that goes to intermediate use or processing. The proportion of the i^{th}

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[3]Dorfman, Samuelson, and Solow, Linear Programming and Economic Analysis, pp. 204-210, McGraw Hill Book Co., 1959

	INTERMEDIATE USE		FINAL DEMAND	TOTAL OUTPUT
	INDUSTRY 1	INDUSTRY 2		
INDUSTRY 1	x_{11}	x_{12}	C_1	X_1
INDUSTRY 2	x_{21}	x_{22}	C_2	X_2
LABOR	x_{01}	x_{02}		X_0

FIGURE 1

industry's output that goes to the j^{th} industry to total output of the j^{th} industry is then

$$\frac{x_{ij}}{X_j}$$

If this ratio is termed as a_{ij}, equation (1) can be written as:

$$a_{i1}X_1 + a_{i2}X_2 + C_1 = X_i \qquad (2)$$

using the proportionality relationship. For each of the two industries, equation (2) is equivalent to the following set of equations:

$$(1 - a_{11})X_1 - a_{12}X_2 = C_1$$

$$-a_{21}X_1 + (1 - a_{22})X_2 = C_2$$

For computational convenience, this set of equations for the two-industry illustration can be converted to a matrix form. If X is the vector of total outputs, C the vector of final consumption or demand, and A the matrix of the a_{ij}'s, the set of two equations can be reduced to

$(I - A)X = C$ where I is the identity matrix, i.e., $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for a two-industry example.

Given a certain level of final demand, the total output necessary to meet this level can easily be derived as

$$X = (I - A)^{-1}C$$

The matrix (I - A) represents the amount of total output needed to generate each unit of final consumption. Since it is derived from the matrix of a_{ij}'s otherwise known as the technology matrix, the inverse of (I - A) or production matrix

must always exist for the problem at hand to be meaningful. In effect, this is what the Hawkins-Simons condition strives to ensure. The ability of a sector to produce final goods depends on whether its total or gross output exceeds the direct and indirect input requirements of that good itself. In other words, to produce one unit of a commodity, the direct and indirect input requirements of that good itself must not exceed one unit.[4]

A graphical illustration for the two-industry model is presented in Figure 2 to lend the reader an intuitive understanding of the Hawkins-Simons condition. Since the available output cannot be less than the sum of its alternative uses, but could even be greater, the following relationships should hold:

$$X_1 \geqq x_{11} + x_{12} + C_1$$

$$X_2 \geqq x_{21} + x_{22} + C_2$$

$$X_0 \geqq x_{01} + x_{02}$$

where the subscript 0 refers to the primary input labor. Using the ratio's or a_{ij}'s earlier derived, the following equations should also hold:

$$(1 - a_{11})X_1 - a_{12}X_2 \geqq C_1 \qquad \text{(L1)}$$

$$-a_{21}X_1 + (1 - a_{22})X_2 \geqq C_2 \qquad \text{(L2)}$$

$$a_{01}X_1 + a_{02}X_2 \leqq X_0 \qquad \text{(L3)}$$

[4]Chio-shuang Yan, <u>Introduction to Input-Output Economics</u>, p. 36, Holt, Rinehart and Winston, 1969.

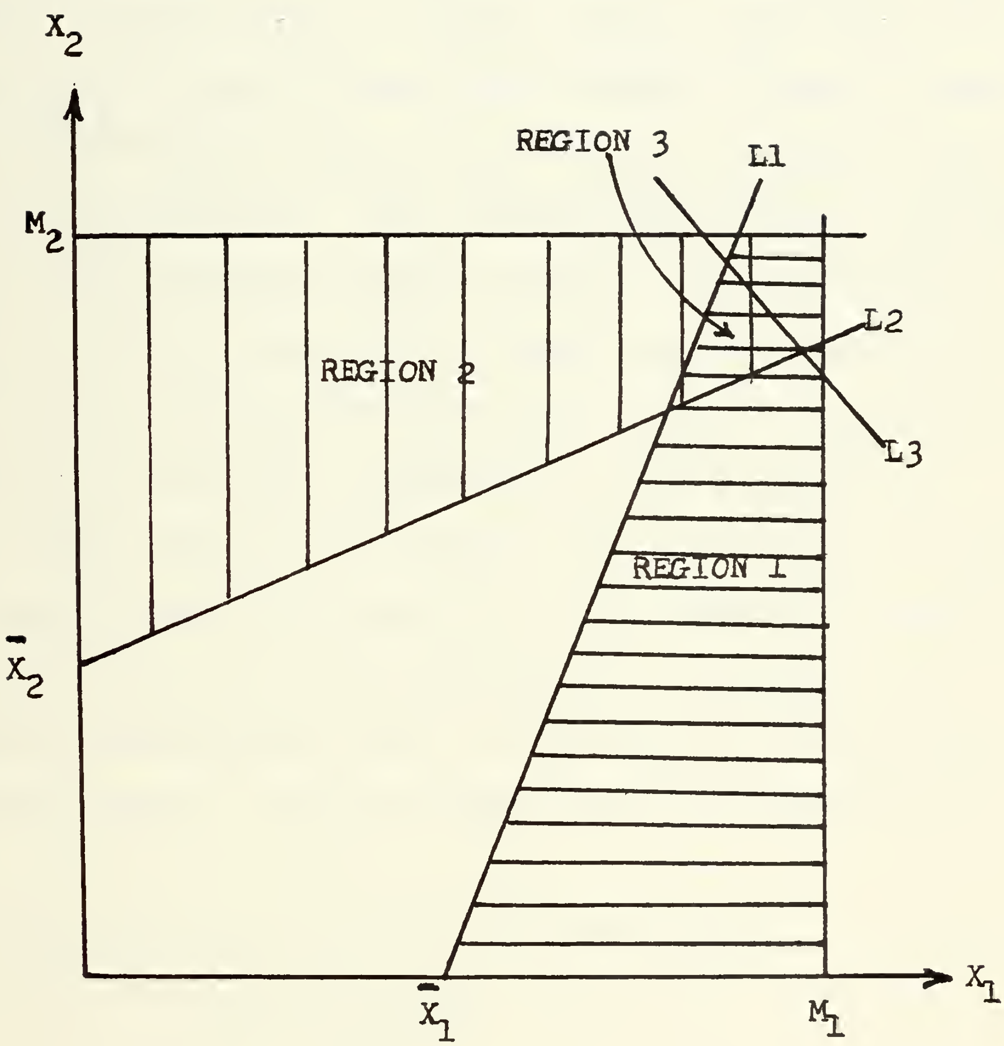

FIGURE 2

L1, L2, and L3 are drawn on a positive quadrant in Figure 2 (since negative total outputs have no meaning if X_1 and X_2 are drawn as the horizontal and vertical axis respectively). M_1 and M_2 are the respective capacities of Industry 1 and Industry 2 respectively. Clearly, Region 1 satisfies the inequality from which L1 had been derived. Region 2 satisfies L2; and Region 3 which covers parts of Region 1 and Region 2, satisfies L3. The cross-hatched Region 3 indicates gross outputs which will yield both the final consumption levels C_1 and C_2. The point of intersection of L1 and L2, which is P, indicates the compatible values of X_1 and X_2 which can provide final consumption levels C_1 and C_2.

If the slope of L2 is bigger than that of L1, or if the two lines are parallel, there can be no point of intersection P for the lines at a positive quadrant. Therefore, the preceding inequalities cannot be satisfied for meaningful positive outputs. No final demand would be producible at all.

The slopes of L1 and L2 are respectively:

$$S_{L1} = \frac{1 - a_{11}}{a_{12}}$$

$$S_{L2} = \frac{a_{21}}{1 - a_{22}}$$

In this two-industry illustration, the condition that should obtain in order that P, the point of intersection exists is that the slope of L2 be less than the slope of L1. This means that:

$$S_{L2} < S_{L1}$$

i.e., $$\frac{a_{21}}{1 - a_{22}} < \frac{1 - a_{11}}{a_{12}}$$

or $$(1 - a_{11})(1 - a_{22}) - a_{12}a_{21} > 0$$

In determinant form,

$$\begin{vmatrix} 1 - a_{11} & -a_{12} \\ - a_{21} & 1 - a_{22} \end{vmatrix} > 0 \quad (3)$$

Inequality (3) assures only that the two lines intersect. To be meaningful however, the point of intersection P should always lie in a positive quadrant. Only positive values of gross output make economic sense. If a condition that implies positive X_1 intercept for L1 and positive X_2 intercept for L2 is not imposed, the intersection at the positive quadrant is not assured. It can be shown that these intercepts depend on the value of $(1 - a_{11})$ and the value of $(1 - a_{22})$. To satisfy the requirement for positive values of the point of intersection P, the following relationships should be met in addition to inequality (3):

$$\begin{aligned} 1 - a_{11} &> 0 \\ 1 - a_{22} &> 0 \end{aligned} \quad (4)$$

The interpretation for inequalities (3) and (4) is that all sub-groups of commodities should be self-sustaining directly and indirectly.[5]

[5] Dorfman, Samuelson and Solow, pp. 212-215

can be expressed into some quantifiable terms, military forces can therefore fit into the framework of input-output analysis.

The following discussion concerns the assumptions of Leontief's input-output analysis and their relevance to the modeling of the flow of support in the Philippine Army:

1. Fixed Coefficients of Production

In developing his concept of input-output analysis, Leontief assumed that it takes a fixed proportion of input to generate a unit measure of an industry's product output. If there are two products that are required as inputs to a productive process, a fixed proportion of each product to the total output of the productive process will be required at all output levels. With this hypothesis, it is implied in the model that there is constant returns to scale. Fixed coefficient of production also implies the non-substitutability of inputs and the absence of technological change.[6]

Constant returns to scale is an acceptable and standard assumption in present-day economics, especially in a short-run context.

Non-substitutability among inputs can be analyzed in terms of the effect of prices of factors. It can be argued that in the real world, industries have alternative input-mixes. Labor and capital are typical cases of substitutable inputs. However, it can be shown here that the proportions of inputs in a productive process will remain the same even

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[6]Carl F. Christ, Input-Output Analysis: An Appraisal, pp. 139-141, Princeton, 1955

In this paper, the Leontief static open input-output model will serve as the basis for the cost methodology to be formulated. A model is static in the sense that the dynamic interplay of one time period or another is not acknoqledged. It is open in the sense that the output of some activities or the final users is not measured in the model.

C. ASSUMPTIONS AND APPRAISAL OF LEONTIEF'S INPUT-OUTPUT ANALYSIS

The first comprehensive input-output model constructed by Leontief was a representation of the United States economy. In fact, most of the modeling done employing input-output analysis as a fundamental tool are related to national and regional economies. Consequently, critics of the concept have centered their evaluation on the economic implications of the assumptions made in formulating an input-output model. An analogy between the industrial sector in an economy and support organizations in the military establishment can be established. The tactical and other non-support forces are beneficiaries of the support, or output if it may be called, provided by the support organizations. By their nature, support organizations also require the output of one another. Support organizations can be classified in the same way as industries or producers are in an economy. Tactical and other non-support forces are the final users because they do not generate any support output, but rather receive or "consume" them only. If the output of support organizations

if prices change as long as their relative values stay approximately constant.

In Figure 3, the production function of the Leontief fixed coefficient case is shown with the classical production function. Roy Harrod argued that the classical production function that can be assumed as homothetic and homogenous of degree one can successfully approximate the Leontief isoquant.

As used in Harrod's argument, homogenous of degree one refers to the constant-returns-to-scale property of a production function, while homothetic pertains to the linear property of the expansion path which joins the points of tangency between budget lines and isoquants, as shown in Figure 3.

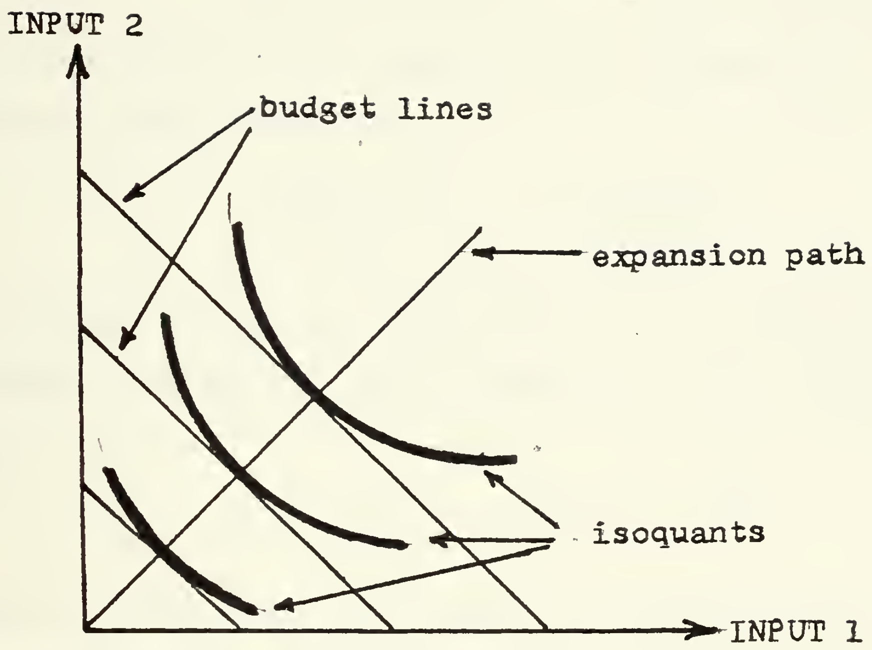
The condition for Harrod's argument is that relative prices stay approximately constant. In the real world, this condition can have basis. Though prices change, their relative values with respect to one another remain approximately the same in most cases.

To show the effect of relative prices to the proportions of inputs in a productive process, the Cobb-Douglas production function will be examined below.

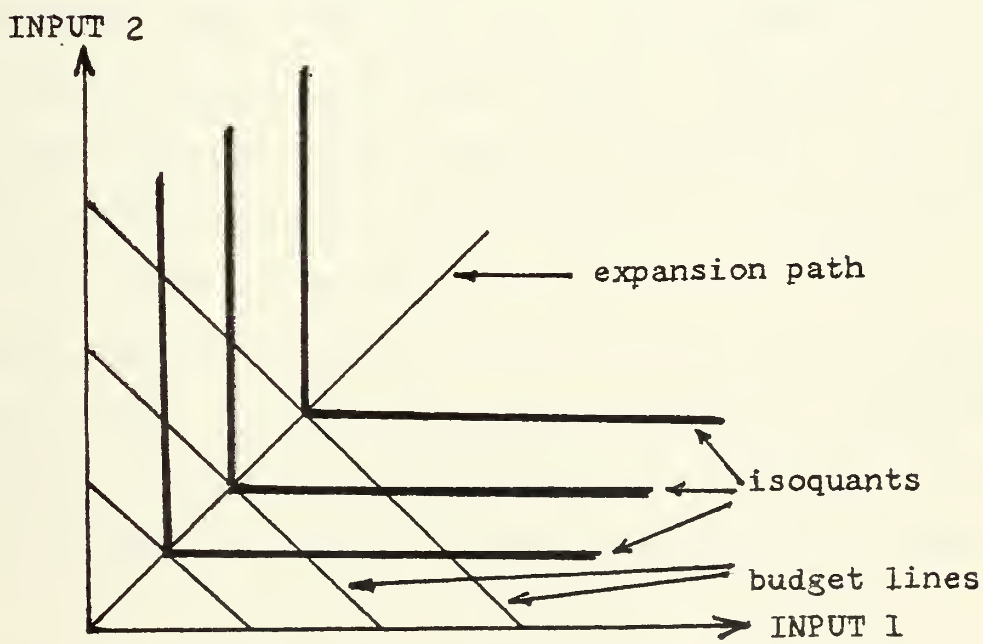
The Cobb-Douglas production function is expressed in the following relationship:

$$Q = K^{a}L^{1-a}$$

where Q is the total output, L, the input labor, K, the input capital, a, the share of capital on income, and (1-a), the share of labor on income. The marginal products with respect to capital and labor are as follows:



CLASSICAL PRODUCTION FUNCTION



LEONTIEF'S PRODUCTION FUNCTION

FIGURE 3

$$\dot{Q}_K = aK^{a-1}L^{1-a} \quad , \quad \dot{Q}_L = (1-a)K^aL^{-a}$$

However, the first order condition for minimization using the Lagrange-multiplier method is:

$$\frac{aK^{a-1}L^{1-a}}{r} = \frac{(1-a)K^aL^{-a}}{w}$$

where r is the return on capital and w is the wage rate in the economy. From this relationship, K can be derived as follows:

$$K = \frac{a}{(1-a)} \frac{w}{r} L \quad (5)$$

Equation (5) shows that the amount of each input will remain in the same proportion if the ratio of prices stays the same. Considering the general behavior of prices in an economy, as noted earlier, their relative values stay approximately constant even if their actual values change. There are few cases where relative prices change significantly. With the arguments presented above, the assumption of non-substitutability due to prices can hold.

Technological change is a matter that cannot be realized in a short time. Although technology continually progresses, its effects, in general, are spread over a long time. Adjustments of existing productive processes are gradually developed.

In the Philippine Army, the case for ignoring the effects on the input coefficients of substitution and technological change is strong. A supported organization requires specific type of support output from a support organization. Generally,

each support organization provides only one type of support output to the other military units. There is no duplication in support function. A support output from an organization can not be substituted by any other output from the rest of the support sector. Substitution can therefore be ignored. Technological change, on the other hand, is not totally non-existent. The Philippine Army belongs to a developing country which is not involved in any military confrontation with any foreign country. In fact, only a small portion of its gross national product is devoted to defense purposes. While modern weaponry is also introduced, the level or degree of sophistication of such weapons does not require a significant alteration of the presently existing support establishments. It is unlikely that weapon systems that can change the present profile of the support structure will be introduced in the near future. The case might have been different if the interest in this study is either the Air Force or the Navy of the Philippines, where the support sector is more susceptible to change in technology because of the nature of their equipment.

Leontief's fixed coefficient production function was shown to correspond to the classical production function based on Harrod's argument. The analysis of the Cobb-Douglas production function indicated that proportions of inputs to a productive process will remain fixed if a certain condition is satisfied. In both of these arguments, the condition that must be met is that the ratio of input

prices remains at a constant value. It would be hard to assume that the ratio of prices will remain at exactly the same level. However, if the change that might have occurred is not significant enough to radically alter the values of relative prices, it is safe to assume that the ratio of input prices is approximately constant. This is especially true if the measures are confined to a narrow time frame. In that case, the effect of price substitution on the coefficients can be considered as negligible. The behavior of prices in the Philippines is generally similar to most Of the capitalist countries. Inflation leads to increase in prices. Increase in prices though affects most, if not all, of the goods in the market. In effect, the relative prices remain stable.

In his appraisal of Leontief's input-output analysis, Hatanaka also considered factors that are outside the model, which can affect the coefficients of production. He noted capital stocks as one of these factors. The amount of capital stocks that exist may cause the input coefficients to be dependent upon the level of output, especially if capital stocks are a bottleneck in production.[7]

In the military establishment, capital stocks would correspond to the capacity of support organizations, which in many ways is determined by the capital investment in the organization. The maximum amount of support output an

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[7]Hatanaka, Michio p. 50

organization can provide may be limited by the existing facilities, beyond which, new investment to increase its capacity have to be provided. However, if the capacity of a support organization is not exceeded, there can be no effect on the input coefficients.

Aggregation was extensively discussed by Hatanaka in his appraisal of input-output analysis. The numerous industries obtaining in an economy model necessitate aggregation of similar or related industries to reduce the productive sector to a manageable proporation. Hatanaka argued that even if individually each industry has a constant input coefficient, they may have input coefficients that are not constant when aggregated.[8]

The number of support organizations in the Philippine Army are not many, to require the level of aggregation implied by Hatanaka. In fact, modern computers can easily handle the computational requirements of an input-output model designed for the Philippine Army without any aggregation. At most, there can only be twenty support organizations to be considered at the level of detail appropriate for analysis from the Army headquarters level. If ever aggregation is to be adopted, it can be confined to the identical Station Hospitals that have separate organizations to support.

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[8] Hatanaka, Michio, pp. 53-55

2. No Joint Production

The nature of Leontief's input-output analysis in an economy setting forces the categorization of industries producing multiple products into the classification framework of industries having only one product. Hatanaka said there is no guarantee that the changes in transactions resulting from this classification will not affect the input coefficients, which would otherwise be constant if joint production were explicitly allowed in the framework of input-output analysis.

The assumption that each productive sector has only one product is valid in the case of the Philippine Army. It is a fact that each support organization has only one identifiable output. Supply, for example, is the only support output of the Army central supply agency.

To sum up this discussion on the assumptions of input-output analysis, while there is ground to contradict the assumptions of the model in an economy setting, its application to the military establishment shows enough reason to believe that the assumptions will hold, especially if adopted to the modeling of the flow of support output in the Philippine Army.

D. AREAS OF USE POTENTIALLY IMPORTANT TO MILITARY COST ANALYSIS

Several areas exist where input-output analysis has been adopted and found valuable in the solution of interindustry problems. Evans and Hoffenberg cited production requirements analysis, forecasting, price equilibrium problems,

and industrial mobilisation problems as some of the economic matters where input-output analysis has been applied.[9] Miernyk stated structural analysis, impact or multiplier analysis, feasibility tests and sensitivity analysis.[10]

The first two uses are potentially important to cost analysis in the military, and the discussion on the varied uses are confined to these two areas below:

1. <u>Production Requirements Analysis</u>[11]

Specific production requirements to meet a specified amount and kind of end-product is the main interest in inter-industry research. Production requirements problems in the simplest form may involve only the processing sector in the economy. For example, one problem might be concerned with the additional output that must be required from each productive sector to support exactly an increase of a single unit in deliveries to the final consumers. In the resulting computation, the outcome would show that continued delivery outside the intermediate sector of an additional unit of output from a given producer is permissible if additional production is generated not only at the given producer but also in all producers called on directly or indirectly to

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[9]W. Duane Evans and Marvin Hoffenberg, <u>The Nature and Uses of Interindustry Relations Data and Methods</u>, Input-Output Analysis: An Appraisal, Vol. 18, pp. 92-114, Princeton University Press, 1955

[10]William H. Miernyk, <u>The Elements of Input-Output Analysis</u>, pp. 30-57, Random House, New York, 1967

[11]Evans and Hoffenberg, pp. 92-93

supply needed materials and services. The increases that are needed are established quantitatively by the computation.

It is readily apparent that the specific impacts throughout the economic system of a particular and specific change in demand can be anticipated. Extension of the analysis can consider not only unit changes for single sectors, but a complete schedule of changes in production by all the producers.

2. Forecast Models

Input-output models have received a due amount of attention in forecasting. Speculations about the future have a ready market. Despite some poor results in forecasting, interests have not been shaken because of contemporary actions that must be conditioned by, and based on expectations for the future.[12]

For the short term forecasts - for the periods of two to three years, it is fairly safe to assume that the input coefficients will not change, or that they will not change significantly.[13]

Consistent forecasting as a term has been applied to the projections of a transactions table. When an input-output table is projected, the output of each industry, according to Miernyk is consistent with the demand, both final and from other industries, for its products. He added though that a consistent forecast is not guaranteed to turn out right. What it does is to insure that the projections for

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[12] Ibid., pp. 104-115

[13] Miernyk, p. 33

individual industries and sectors will add up to a total projection if the structural relations of the economy do not change significantly over the projection period, or if allowance can be made for anticipated changes in the structural relations.[14]

Miernyk also noted that the accuracy of inter-industry projections will depend on the accuracy with which the final demand projections can be made. Even if there is a certain amount of errors in the projections of final demand as may be expected, the resulting projections of inter-industry transactions will be useful to economists, business analysts, and policy-makers.[15]

One thing was stressed by Miernyk: that forecasting should be limited to short-run projections. This is so because the model is static in the sense that it assumes no change in the input parameters. He said that small changes that might occur over a relatively short period of time would not lead to serious errors in the projected transactions table.[16]

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[14] Ibid., pp. 32-33

[15] Ibid., p. 37

[16] Ibid.

III. COST ANALYSIS AND A ROLE FOR INPUT-OUTPUT TECHNIQUES

There are recurring problems typical in a military organization which require cost information. Augusta and Jenner cited three areas where internal management problems occur: the determination of optimal alternative force structures, the construction of effective plans, and the evaluation of alternative equipment and systems.[17]

In structural changes, the optimum size and components of the organization are the most common object. Re-evaluation may be necessary too when new equipment which can radically change either the organization's capability or the support requirement is introduced. Reassessment of the structural aspects of the organization could therefore be a continual activity.[18]

In building effective plans, detailed cost information could be one of the essential needs. Suppose plans are being prepared in connection with the reduction of forces in a certain operational area. The indirect or derived effects of force contraction on support organizations outside that operational area must be considered. Furthermore, the cost of alternatives must be derived quickly to minimize the required time for planning.[19]

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[17] Ibid.

[18] Ibid.

[19] Ibid.

Cost-effectiveness studies in alternative equipment and systems also require cost information. To the of value, such studies should include all direct and indirect costs of the alternatives. In this way, the decision-maker can be assured that an important aspect bearing on the case at study, i.e., the implicit costs, is fully considered.[20]

Aside from internal management problems, Augusta and Jenner also cited external requirements for cost information. In the United States Department of Defense, for example, requests for data from the services are many because of their integrated Planning, Programming, and Budgeting System. The information on cost feed into current budgetary decisions, thus making timely and accurate cost estimates crucial.[21]

A. THE CENTRAL PROBLEM IN COST ANALYSIS

The value of benefits lost is cost. Both costs and benefits are the consequences of selected alternatives. To identify the logical choice among a set of alternatives, each one should be analyzed and compared with the other possible selection. The task of the cost analyst, according to Fisher in his <u>Cost Considerations in Systems Analysis</u>, is to identify and to measure and evaluate the benefits foregone by choosing one course of action rather than another. One way of providing an indication of cost to the decision-maker is by enumerating the required resources to pursue a particular course of action.

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[20] <u>Ibid</u>.

[21] <u>Ibid</u>.

Another way is to determine the alternative uses of such resources, or to estimate the value of these alternative uses.[22]

The central problem that faces the cost analysis, said Fisher, is the development and application of the concepts and techniques for assessing the economic cost of proposed alternative future actions under conditions of uncertainty. Usually, the alternative actions in national security problems are in the form of one or combinations of the following:[23]

1. Proposed new capabilities for the future, like a new weapon system.

2. Proposed new modifications of existing or presently programmed activities.

3. Proposed deletions from presently planned force.

4. Proposed combinations or packages of 1 through 3, i.e., total force structure or major subsets of total force structures for the futures.

B. TYPICAL OUTPUTS OF COST ANALYSIS STUDIES[24]

Assuming that the output-oriented package of military capability is a weapon system, or its equivalent, and that force structures are made up of combinations of these packages, Fisher enumerated four types of contests of

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[22]Gene H. Fisher, Cost Considerations in System Analysis, p. 62, American Elsevier Company, New York, 1974

[23]Ibid., pp. 64-65

[24]Ibid., pp. 82-98

military cost analysis that can be considered. They are intrasystem comparisons, intersystem comparisons, force-mix comparisons, and total force structure cost analyses.

In intrasystem comparisons, the primary emphasis is on explorations of how cost varies as the configuration of the proposed system is changed. One form of intrasystem cost analysis is total system cost as a function of force size for varying number of years of operation. The resulting trade-off information from such explorations can be very useful in reaching judgment about the optimum configuration of the system.

Two or more systems are involved in intersystem comparisons. Having the same characteristic or purpose, the systems are evaluated for suitability or desirability in relation to the goals of the decision-maker.

Comparisons of alternative force-mixes in a projected deployment of forces comprise an important subset of the total spectrum of problem areas in system analysis studies. In any of the cases under this analysis, complementarity among the alternative modes is the key factor.

Fisher claims that no one has yet been able to devise ways of quantitatively assessing the effectiveness of total force structures. Nevertheless, he said, the cost aspect of total force planning is not an infeasible project to pursue. A well rounded cost analysis capability must include the ability to investigate the cost implication of alternative future total force proposals. This capability

must also be able to assess rapidly the resource impact of alternative proposals. The number of calculations involved in total force cost analysis require automation in its entirety or in part.

Input-output analysis in the form of a cost model can be useful in force-mix comparisons and total force structure cost analysis. In developing the level and structure of forces, plans evolved should somehow be consistent with the projected amount of resources to be made available for defense purposes. Because of the complexity of support costs which comprise a major part of total force cost, a quick and fairly accurate estimate of support costs can be of considerable use in developing a comprehensive analysis of the alternative level and structure of forces to adopt. The technique of input-output analysis offers a useful tool in these types of cost analyses.

IV. THE DATA ASPECTS IN THE COST MODEL

In a production model oriented on national economic problems, the use of input-output analytical methods requires the establishment of consistent connections between demand for finished products, on one hand, and the implications of this demand for production, employment, capacity utilization, and resource use levels of industries that may be significantly though remotely involved, on the other.[25] Support cost estimation by way of input-output analysis is similar in many ways to the national economy modeling for which the concept had been originally adopted. It is essential that the quantifiable variables which are the support output in the cost model be consistent with the demand for such output. Like in the national economy model, data collection to measure the inputs and outputs of productive sectors can be a very difficult part in building the cost model. Given substantial preparation time, an analyst involved in such a project may possibly surmount the difficulties. But then, the advantage of rapid estimation obtaining in an input-output cost model may be lost. Considering the time frame of most planning and decision-making processes, the exact measurement deemed desirable may not be possible at all.

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[25]W. Duane Evans and Marvin Hoffenberg, The Nature and Uses of Interindustry-Relations Data and Methods, Input-Output Analysis: An Appraisal, Vol. 18, p. 53, Princeton University Press, 1955

A. THE USE OF PROXY VARIABLES

Recognizing the output of a support organization in the military establishment may not be a difficult problem in building the cost model. In the Philippine Army especially, organizational structuring isolates and identifies the expected support an organization established for such purpose should provide. For example, medical service is provided by medical units, supply by supply units, etc. But measuring the support one such unit provides to the array of recipient military units can be a time-consuming task when the transactions involved are very numerous and detailed. Another difficulty of measuring the actual support output is in finding a common numeraire for the different forms of real output a support unit provides. A hospital, citing an example, obviously provides medical services. These services however, are in forms of in-patient treatment, out-patient treatment, or plain emergency medical service. The selection of a numeraire in this case would not be an easy task. The same problem may come out of several organizations which provide related support output. Unless the real outputs are exactly identical, the choice of a numeraire would pose some difficulties.

Augusta and Hibbs suggest the use of proxy variables.[26] Rather than measuring the flow of actual output, a proxy variable is selected to represent that flow of output to

[26] Augusta and Hibbs, Estimating U.S. Navy Support Costs, pp. 17-18

intermediate users and final consumers. Basically, the idea is to assume that the value of a proxy variable changes proportionately with the inputs to other support organizations and with the consumption of the non-support units. For example, it may be safe to assume that medical support would vary with the number of personnel assigned with a supported unit.

Whenever possible, given the time restrictions, the actual flow of output should be a more desirable basis for establishing the technology matrix. In the case where there may be two or more candidates in the role of a proxy variable, Augusta and Hibbs suggested the choice of the proxy variable which can be obtained at least cost. They also suggested that proxies should be selected with careful thought, making sure that such proxies are reasonable. They implied also that the same proxy variable cannot be used to estimate the flow of output of two or more support organizations which appear as separate items in the transactions matrix. Otherwise, identical rows would appear in the matrix, leading to serious problems in matrix inversion.

While no quantified approach was explicitly endorsed by Augusta and Hibbs, Leary, Ferri, Mason, and Brady implied the use of statistical techniques in the validation of proxy variables.[27] Correlation analysis was suggested as a procedure in the proper choice of proxy variables, since it

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[27]Leary, Ferri, Mason, and Brady, Examination of Mission Categories of General Support and Logistics, Vol, 1, Summary and Correlation Analysis, Operations Research, Inc., pp. 17-37

is generally used to determine the degree of mutual relationship between two variables. Furthermore, results of correlation analysis can assist in aggregation of activities. It can help assure that activities aggregated have approximately the same degree of positive correlation.

The degree of correlation or closeness of linear relationship between two variables is measured by this term correlation coefficient. It can take on a value between -1.00 and +1.00. If the value of the correlation coefficient approaches -1.00, then the variables are said to be negatively correlated. If the value approaches zero from either side, the variable are said to be uncorrelated. Positive correlation obtains when the value approaches +1.00. What is deemed proper in the selection of a proxy variable is positive correlation which is significantly different from zero.[28]

If x_i and y_i represent the i^{th} paired observations for two variables X and Y, the correlation coefficient r for N number of paired observations is determined as follows:

$$r = \frac{\frac{1}{N}\sum_i x_i y_i - \bar{x}\,\bar{y}}{\sqrt{\left(\sum_i \frac{x_i^2}{N} - \bar{x}^2\right)\left(\sum_i \frac{y_i^2}{N} - \bar{y}^2\right)}}$$

where $\bar{x}$ and $\bar{y}$ are the average values of X and Y respectively.[29]

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[28] Ibid.

[29] Audrey Haber and Richard P. Runyon, General Statistics, pp. 133-139, Addison-Wesley Publishing Co., 1969.

To illustrate the correlation analysis technique in the validation of a proxy variable, the following hypothetical case is presented. Suppose that the number of assigned or issued tracked and wheeled vehicles in each organization is being considered as a possible proxy variable to represent the output of the Maintenance Battalion. Correlation analysis is then conducted to check the validity of the choice. The following set of paired observations was gathered to calculate the correlation coefficient:

UNIT	NUMBER OF JOB ORDERS PERFORMED	NO. OF VEHICLES
1st Div	330	250
2nd Bde	150	110
3rd Div	200	210
4th Div	280	240
5th Bde	100	106
HHSG	50	80
51st Engr	250	200

Using the expression for the correlation coefficient previously introduced will yield a value of 0.9592 which indicates a high positive value. The number of vehicles may therefore be used as a proxy variable to represent the flow of output of the Maintenance Battalion.

One question that might be raised is why go through the trouble of finding a good proxy variable when the actual output is in fact measured in the testing procedure. There might be a need to update the coefficients of the technology matrix from time to time. Actual measurement of the output can be a time consuming task an analyst is constrained to avoid. Rather than measuring the actual output by examination of all relevant records of the different organizations,

a proxy variable measurement obtainable at lesser cost in time and effort can provide the necessary updates at close approximation.

What is desired in the choice of a proxy variable is one which is positively correlated with the measure of the actual output at a value significantly greater than zero. To check the validity of a choice, statistical hypothesis testing may be used, as suggested by Leary, Mason, Ferri, and Brady.

Assuming a bivariate normal distribution for the proxy and actual variables, the test statistic would be as follows:

$$t = \frac{r\sqrt{(n-2)}}{\sqrt{1-r^2}} \qquad (1)$$

This is Student t distributed with n-2 degrees of freedom.[30]

In the example illustrated previously where the calculated correlation coefficient r is equal to 0.9592 at seven paired observations, the test statistic value from equation (1) above would be 7.586. If the null hypothesis is that the correlation coefficient is equal to zero, or H_0: r = 0, and the alternative hypothesis is that this coefficient is greater than zero, or H_1: r 0, at a confidence level of 99.5%, the critical value for t at five degrees of freedom is 4.032. The test therefore rejects the hypothesis that the correlation coefficient calculated is equal to zero.

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[30] Gerhard Tinter, <u>Mathematics and Statistics for Economists</u>, p. 287, Holt, Rinehard and Winston, New York, 1953

B. FIXED COSTS AND THEIR TREATMENT

When costs are derived from an input-output model they would represent costs that vary proportionately with the level of demand of the final users. If P represents a row vector of prices, the costs associated with a given level of total output X, which is a column vector, can be expressed in the following relationship:

$$\text{Costs} = PX$$

The total output X, however, is equal to $(I - A)^{-1}C$, as derived in the second chapter. Hence Costs can be expressed as follows:

$$\text{Costs} = P(I - A)^{-1}C$$

In many cases, support organizations incur some expenditures independent of the level of forces or final consumers. To cite an example, a central supply agency would incur the same normal installation expenditures like janitorial services, water and electrical consumption, and such other related matters independent of the level of forces supported. What is therefore implied is the existence of fixed costs.

In determining cost estimates related to force structure, it is frequently desired to predict total costs for budgetary reasons. The estimate provided by the input-output model is a major portion of the costs of the planned activities, but it does not account for fixed costs. These fixed costs imply non-proportionality which must be dealt with.

A problem related to fixed costs is the capacity constraint which determines the maximum amount of output support

organizations can produce. Large increases in force size might require new support facilities with different operating costs aside from the initial investment costs that must be provided.[31]

Input-output-analysis is a marginal type of analysis. If large changes are introduced in any part of the model, discontinuities may occur which are beyond its scope.

Johnston noted that the best source of information for the short-run analysis would be records of firms (in an industrial setting) over successive periods of time during which their capacity had remained unchanged.[32]

When no significant alteration is done on the capital structure of the support organization, a stable approximation of the fixed costs incurred can be obtained.

There are two approaches to the problem of fixed costs estimation that may be considered here. An accounting approach could be adopted to identify the costs which remain fixed whatever level of forces is supported. Records of expenditures maintained in an organization may show these items the analyst would be interested to examine. Personnel assigned in a support unit may be well-aware of such cost categories. Another approach that might be pursued is a time series method of estimating that fixed portion of a support unit's expenditures. Basically, the idea is to

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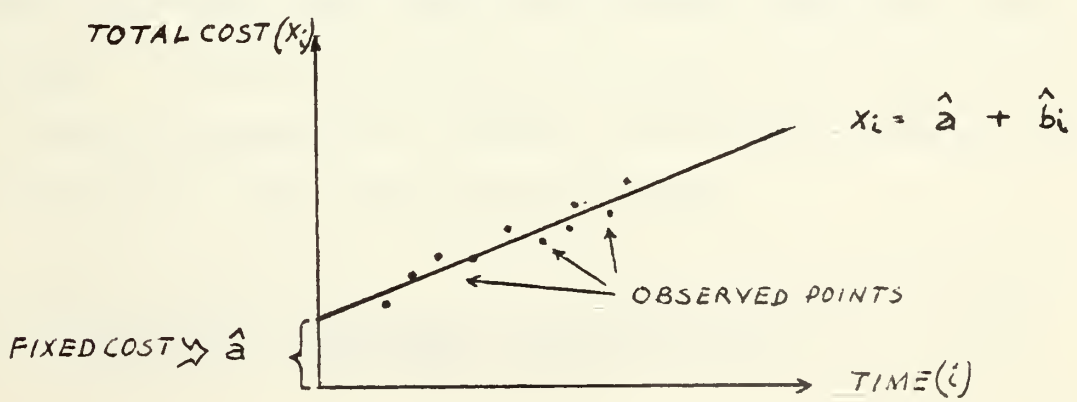
[31] Augusta and Hibbs, p. 22

[32] J. Johnston, Statistical Cost Analysis, p. 28 McGraw-Hill Book Co., 1960

plot the values of the total cost incurred over several time priods, and fit a linear equation by way of simple regression to estimate the vertical intercept on the cost axis.

Since a linearity assumption is again taken as a recourse in this model, it may be worthwhile to paraphrase what Augusta and Hibbs said about this matter. In citing the inevitable challenge that a linear assumption would attract, they asserted that they have never had anyone come forward with an emperically determined and non-linear cost function nor even a cost function with a non-zero intercept. They also referred to the comments from Paul McClenon who, in his <u>A Study of the Electric FYDP With Preliminary Comments on its Use in the Air Staff</u>, maintained that despite the ready challenge on the linearity assumption, analysts usually accept it for recurring costs.[33]

Below is the graphical representation of the time series approach. Total cost x_i is plotted versus time period $\underline{i}$. The estimated fixed cost will be the intercept a on the vertical axis of the estimated regression line.



[33]Augusta and Hibbs, p. 23

To estimate the parameters of the regression line

$$\hat{x}_i = \hat{a} + \hat{b}_i \quad ,$$

the following relationships are used:

$$\hat{b} = \frac{\sum_i (i-\bar{I})(x_i-\bar{x})}{\sum_i (i-\bar{I})^2} \quad , \quad \hat{a} = \bar{x} - b\bar{I}$$

where $\bar{I}$ and $\bar{x}$ are the means of the i's and x's respectively.[34]

The second approach might seldom be applied, if at all. Time series analysis, which is commonly a long-run technique in estimation, may be hard to reconcile with the short-run nature of analysis implied in the basic input-output model. Capacity or the capital structure of the support organization should remain constant in the context of a short run prediction. Then, there is the challenge of finding a good fit for the line that estimates the true relationship between the dependent and independent variables. Few point or observations may not be able to accurately depict the true relationship because of the potential effects of outliers in the scatter plot. It may be necessary to incorporate more data points; and that would mean getting more historical observations. It would be difficult to assume that the capacity or capital structure of a support organization remained unaffected for a very long period of time.

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[34] Jan Kmenta, Elements of Econometrics, pp. 206-208, The Macmillan Co., New York, 1971

This writer would suggest the use of the accounting approach because it would seem less sensitive to the effect of changes in capacity or capital structure of a support organization

C. MEASUREMENT OF SUPPORT OUTPUT

The output of each support organization in the Philippine Army, as suggested by this writer based on personal observations, are listed below for use in the cost model:

1. Quartermaster Battalion

This organization is responsible for the procurement and issuance of supply items required in the different organizations of the Army. Requisition and Issue Vouchers from the units reflect the peso cost of supplies distributed. Since these transactions are journalized for every supported unit, it is suggested that the output be measured in terms of peso or monetary cost of supplies provided. Potential proxy variables for this output are number of personnel assigned, number of major armaments including individual weapons, number of major organizational equipment, and number of company-size sub-units in the supported organizations.

2. Maintenance Battalion

Third echelon maintenance or repair of major armaments and organizational equipment is performed by this support unit for the different units in the Army. There are records which show the job orders done for the supported units.

Since the types of repair work are confined to third echelon maintenance, the output to be measured is suggested to be the number of job orders. As proxy variables, the following data from the supported units may be tapped as possible proxy variables: number of personnel assigned, and number of individual and crew-served weapons issued.

3. Signal Battalion

Communications in the Army is handled by the Signal Battalion. Line communications cost, i.e., telephone services cost, remains approximately fixed. Cost of radio communications vary though. Repairs of radio sets are also performed in this unit. As output, the number of messages (except via telephone) received from and transmitted to the different major units of the Army is recommended to be measured. The consumption of the Signal Battalion itself would be the messages handled involving parties outside of the Army, like the Navy and Air Force. Potential proxy variables are the number of radio sets, and number of company-size sub-units in each of the units supported.

4. Army School Center

Service schooling for the officers and enlisted personnel of the Army is the responsibility of this support unit. Output suggested to be measured is the number of personnel trained from each of the supported units. As proxy variable, the number of assigned personnel in each of the supported units can be considered.

5. Army Finance Center

This organization is primarily concerned with the pay

and allowances of personnel and other related financial matters. The recommended measure of support output is the number of payrolls and vouchers processed from the units supported. Possible proxy variable is the number of company-sized sub-units in the organizations supported.

6. Transportation Battalion

The major support provided by this organization is the transport of supplies and personnel. Vehicle-mile measure of missions performed for the other units in the Army can be derived from record of dispatches. Vehicle-mile could therefore be the unit of measure of its support output. Possible proxy variable is the number of personnel assigned in each of the supported units or distance from the base of this battalion to the bases of the different units in kilometers.

7. Station Hospitals

There are several Station Hospitals located in the camps or bases of major Army units. Generally, medical cases handled are those not requiring extensive treatment or hospitalization, (which are referred to the Medical Center that has responsibility over such cases and is under the functional supervision of General Headquarters, Armed Forces of the Philippines). It is suggested that the number of cases handled during the period under consideration be taken as the measure of output. Possible proxy variables are the number of personnel assigned, and the total number of dependents of personnel in each of the supported units.

8. Special Services

Morale-boosting activities are the primary responsibility of this organization. As numeraire for its output, the monetary cost of services provided and recreational items issued is suggested. If the measurement of its output poses some difficulty, a possible proxy variable is the number of recreational facilities in the base of each supported unit.

In using the suggested proxy variables above, only one type of support output can be represented by a particular proxy variable. For example, if number of personnel assigned in each of the organizations is selected to represent the output of the Army Finance Center, the same proxy variable cannot be used anymore to represent some other type of output. It should be noted also that the suggested proxy variables are not necessarily the only candidates. They are only the conceived proxy variables at the time of this writing. Further research and statistical verification may yield some other proxy variables, especially if done in the Philippines.

V. AN APPLICATION OF INPUT-OUTPUT ANALYSIS IN MILITARY COST MODELING

In this chapter, an illustrative example of the application of input-output analysis is presented. The data utilized in this illustration are hypothetical in nature. Difficulties encountered in obtaining actual data relevant to the cost model had compelled this writer to take this recourse at data generation. However, experience has guided this writer in formulating the relative values of inputs and outputs presented in this example. The data necessary in building the cost model are obtainable in the Philippines should this technique be applied there. The organizational structure shown here is not exactly the one presently adopted in the Philippine Army. There is however, a close semblance between the one presented and the existing organizational structure.

With support units assuming the role of producers or industries in a Leontief static open input-output model, the methodology using input-output analysis can be formulated.[35] The final users are the supported forces, which are mostly tactical or strategic forces. Since support costs are implicit by nature of their relationship with the combat forces, an input-output methodology could be an appropriate approach in determining the impact of structural changes in combat forces.

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[35] The methodology and notations here follow substantially the NARM model of Augusta and Hibbs cited earlier.

The level or structure of tactical and strategic combat forces is one common area of interest in military planning, hence the application of input-output analysis could be wide and extensive.

To establish the methodology for support cost estimation, it is necessary to trace the flow of support from each of the support units to other support units, and to the tactical or non-supporting units.[36] The distribution of primary inputs or exogenous factors to the support and non-supporting organizations should also be incorporated in a tableau to allow the estimation of exogenous resources related to a given force level.

Figure 4 illustrates the features of a tableau that will serve as a basic foundation in the cost estimation methodology.[37] The flow of support output from each support unit to the other support organizations will be contained in the U matrix. If there are N support units, U will be a square NxN matrix. V is the matrix of support output flow from support units to the purely supported units or final consumers - the tactical forces and other non-support units in this case. If there are M tactical units, V is an MxM matrix. Exogenous resources flow are reflected in W and Z matrices. Flow of exogenous resources to support units are

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[36] These distributions comprise the intermediate use and final consumption respectively.

[37] Support, as defined here, excludes the support that combat units like the artillery and armor forces provide to other combat forces in the form of firepower, mobility, and the like.

	SUPPORT UNITS (1) (2) . . .(N)	NON-SUPPORT UNITS (1) (2). . . (M)
SUPPORT UNITS (1) (2) . . . (N)	U NxN	V NxM
BUDGETARY PROGRAMS (1) . : (K)	W KxN	Z KxM

FIGURE 4

contained in W, while flow to tactical units are reflected in Z. If there are K different exogenous resources, W and Z are KxN and KxM respectively.

A. ESTIMATION OF TOTAL SUPPORT OUTPUT GIVEN A NEW LEVEL OF FINAL CONSUMPTION

The relationship $X=(I-A)^{-1}C$ earlier discussed as a predicting device in input-output analysis will be the computational tool in this application. To derive the vector of total output X, let

$$x_i = u_{i1} + u_{i2} + \ldots + u_{iN} + v_{i1} + v_{i2} + \ldots v_{iM}$$

for each row i or for i = 1,2,. . .,N.

To derive the vector of final demand C, let

$$c_i = v_{i1} + v_{i2} + \ldots, + v_M$$

for i = 1,2,. . .,M.

To show how this technique works in practical application, assume that there are eight support units, eight non-support units and two budgetary programs. Figure 5 shows the list of these programs and units. The transaction table that would represent the system is shown in Figure 6, while the entries of the transaction table for the U, V, W, and Z matrices are indicated in Table 1. Following the procedure outlined above, the values for the total output vector X and final consumption C for the example will be:

$$X^t = (17555.0 \quad 2091.0 \quad 204.0 \quad 903.0 \quad 2480.0 \quad 123.5 \quad 80.8 \quad 265.5)$$

$$C^t = (14000.0 \quad 1900.0 \quad 164.0 \quad 815.0 \quad 2305.0 \quad 106.0 \quad 69.0 \quad 227.0)$$

LIST OF SUPPORT UNITS

1. Quartermaster Battalion
2. Maintenance Battalion
3. Signal Battalion
4. Army Finance Center
5. Army School Center
6. Transportation Battalion
7. Station Hospitals
8. Special Services

LIST OF NON-SUPPORT UNITS

1. 1st Infantry Division
2. 2nd Infantry Brigade (Separate)
3. 3rd Infantry Division
4. 4th Infantry Division
5. 5th Infantry Brigade (Separate)
6. Home Defense Forces Group (Airborne)
7. Headquarters and Headquarters Service Group
8. 51st Engineer Brigade

LIST OF BUDGETARY PROGRAMS

1. Personnel Services
2. Operations and Maintenance

FIGURE 5

	SUPPORT UNITS	NON-SUPPORT UNITS
	Quartermaster Bn Maintenance Bn Signal Bn Army Finance Center Army School Center Transportation Bn Station Hospitals Special Services	1st Inf Div 2nd Inf Bde 3rd Inf Div 4th Inf Div 5th Inf Bde HDFG(Airborne) HHSG 51st Engr Bde
Quartermaster Battalion Maintenance Battalion Signal Battalion Army Finance Center Army School Center Transportation Battalion Station Hospitals Special Services	U	V
Personnel Services Operations and Maintenance	W	Z

FIGURE 6

100.0	700.0	300.0	50.0	450.0	50.0	5.0	1000.0
60.0	40.0	25.0	15.0	11.0	9.0	6.0	25.0
5.0	4.0	10.0	7.0	4.0	2.0	2.0	6.0
10.0	10.0	10.0	8.0	25.0	2.0	3.0	20.0
20.0	20.0	20.0	10.0	10.0	5.0	65.0	25.0
2.0	2.0	2.0	2.0	1.5	1.0	6.0	1.0
1.5	1.5	1.5	1.3	1.0	1.0	1.0	3.0
5.0	7.0	6.5	4.0	7.0	2.0	1.0	6.0

U MATRIX

3500.0	1700.0	2500.0	3000.0	1500.0	200.0	500.0	2000.0
400.0	150.0	200.0	500.0	150.0	20.0	180.0	300.0
40.0	10.0	40.0	40.0	10.0	2.0	7.0	15.0
200.0	80.0	150.0	180.0	75.0	15.0	25.0	90.0
600.0	250.0	450.0	500.0	220.0	20.0	35.0	230.0
30.0	8.0	25.0	31.0	6.0	2.0	1.0	3.0
15.0	5.0	11.0	14.0	10.0	1.0	4.0	9.0
50.0	26.0	40.0	45.0	24.0	10.0	7.0	25.0

V MATRIX

3960.0	4100.0	4200.0	3500.0	4050.0	4150.0	6500.0	2000.0
17555.0	2350.0	2500.0	9000.0	1000.0	2500.0	5150.0	350.0

W MATRIX

45000.0	15000.0	40000.0	42000.0	12000.0	9000.0	10000.0	16000.0
5000.0	2500.0	3500.0	4900.0	2300.0	300.0	2000.0	2400.0

Z MATRIX

TABLE 1

where the units of measure for the output of the support units are as follows:

Quartermaster Battalion	:	Cost of issued supplies in thousands of pesos
Maintenance Battalion	:	Number of job orders
Signal Battalion	:	Number of radio messages received/transmitted in hundreds
Army Finance Center	:	Number of payrolls/vouchers in hundreds
Transportation Battalion	:	Number of vehicle-miles in thousands
Station Hospitals	:	Number of medical cases in thousands
Special Services	:	Cost of services and entertainment goods in thousands of pesos

To create the technology matrix "A", each element of the j^{th} column of U is divided by the j^{th} element of the vector X. Notationally, this is equivalent to:

$$\frac{u_{ij}}{X_j} = a_{ij} \text{ for } i,\ j = 1,2,.\ .\ .,N$$

The results of these computations are shown in Table 2.

To the analyst, the focal point of interest in applying this methodology is the $(I - A)^{-1}$ matrix. To derive this matrix, the technology matric "A" is subtracted from the identity matrix I as shown in the results depicted in Table 3. The result of the inverse operation is indicated in Table 4.

Continuing with the sample application, if the final consumption is changed from a level C to a new level C', the new total output X' required to provide for this new final demand is equal to $(I - A)^{-1}C'$. Therefore, if the 2nd Infantry Brigade in the illustrative example is expanded

0.005	0.334	1.470	0.055	0.181	0.404	0.061	3.766
0.003	0.019	0.122	0.016	0.004	0.072	0.074	0.094
0.000	0.001	0.049	0.007	0.001	0.016	0.024	0.022
0.000	0.004	0.049	0.008	0.010	0.016	0.037	0.073
0.001	0.009	0.098	0.011	0.004	0.040	0.804	0.094
0.000	0.001	0.009	0.002	0.000	0.008	0.074	0.003
0.000	0.000	0.007	0.001	0.000	0.008	0.012	0.011
0.000	0.003	0.031	0.004	0.002	0.016	0.013	0.022

TABLE 2
"A" MATRIX

0.994	-0.334	-1.470	-0.0554	-0.181	-0.409	-0.061	-3.766
-0.003	0.980	-0.122	-0.016	-0.004	-0.072	-0.074	-0.094
-0.000	-0.001	0.951	-0.007	-0.001	-0.016	-0.024	-0.022
-0.000	-0.004	-0.049	0.991	-0.010	-0.016	-0.037	-0.075
-0.001	-0.009	-0.098	-0.011	0.996	-0.040	-0.804	-0.094
-0.000	-0.001	-0.009	-0.002	-0.000	0.991	-0.074	-0.003
-0.000	-0.000	-0.007	-0.001	-0.000	-0.008	0.987	-0.011
-0.000	-0.003	-0.031	-0.004	-0.002	-0.016	-0.012	0.977

TABLE 3
(I-A) MATRIX

1.009	0.364	1.775	0.098	0.201	0.545	0.393	3.998
0.003	1.021	0.144	0.019	0.006	0.082	0.094	0.119
0.000	0.002	1.054	0.008	0.002	0.018	0.030	0.027
0.000	0.005	0.058	1.010	0.010	0.020	0.051	0.084
0.001	0.011	0.118	0.014	1.005	0.053	0.828	0.116
0.000	0.001	0.011	0.002	0.000	1.009	0.077	0.005
0.000	0.000	0.008	0.001	0.000	0.008	1.014	0.012
0.000	0.003	0.036	0.005	0.003	0.018	0.018	1.026

TABLE 4
(I-A) INVERSE

to the size of the 1st Infantry Division, the new level of final consumption C' can be derived by subtracting the second column in the V matrix corresponding to the 2nd Infantry Brigade and adding the new level of consumption for the Brigade, which is now equal to that of the 1st Infantry Division, to the old final consumption vector C. This will yield the following values for C':

$$C' = \begin{bmatrix} 16700.00 \\ 2150.00 \\ 194.00 \\ 935.00 \\ 2655.00 \\ 128.00 \\ 79.00 \\ 251.00 \end{bmatrix}$$

The new total output that must be provided by the support units to allow for this new level of consumption will be:

$$X' = \begin{bmatrix} 19709.00 \\ 2367.00 \\ 239.80 \\ 1035.00 \\ 2854.00 \\ 148.06 \\ 92.46 \\ 295.04 \end{bmatrix}$$

(Note: All the figures in the illustration are calculated on a yearly basis)

The same procedure as above could be used if the new level consumption is due to the addition of forces. For example, if an additional Brigade similar to the size and employment of the 2nd Infantry Brigade is to be organized, the new final consumption vector C' would be increased by the amount of consumption of the 2nd Infantry Brigade. The total output vector will be computed as in the original example above.

B. ESTIMATION OF EXOGENOUS RESOURCES REQUIREMENTS RELATIVE TO A GIVEN LEVEL OF CONSUMPTION

The Z matrix indicates the amounts of each budget program funds that go to the final users or the supported units directly. Changes in the level or structure of any of the non-support units are assumed to be proportional to the change in the budgetary program funds. From the previous example on variation in final consumption, where the 2nd Infantry Brigade was expanded to the size of the 1st Infantry Division, the amounts of the two budgetary program funds will assume the values of that the 1st Infantry Division for the second column of the Z matrix. In other words, instead of 15,000,000 pesos of personnel services and 2,500,000 pesos of operations and maintenance funds, the new requirements will be 45,000,000 pesos and 500,000,000 pesos respectively for the 2nd Infantry Brigade.

For the support units, the new level of total output brought about by the change in final consumption causes a corresponding change in the requirements for budgetary program funds. A new matrix B is constructed to estimate this new requirement. Each element b_{ij} of the matrix B indicates the amount of budgetary program _i_ that must be provided to support organization j in order that the latter generate a unit of its output. Thus

$$b_{ij} = \frac{w_{ij}}{X_j} \quad \text{for } \begin{array}{l} i = 1,\ldots,8 \\ j = 1,\ldots,8 \end{array}$$

The resulting matrix B in the illustrative example is shown in Table 5.

If the new total output is X', the composition of budgetary programs that must be made available to each of the support organizations is calculated by multiplying each element in the j^{th} column of B by the j^{th} element of the new total output X'. Thus, the resulting exogenous resource requirements W' of the support organizations will be

$$w_{ij} = b_{ij}X_j' \quad \text{for } i = 1,2 \quad j = 1,2,\ldots,8$$

The new W' matrix is shown in Table 6. Each of the values of w_{ij} increased as a result of the increase in total support output required to meet the new level of final consumption. These figures indicate the increased indirect costs of a change in the level of forces which input-output analysis can account.

The figures in the matrix of flow of exogenous resources to support organizations represent only the variable portions of cost. In generating the data for this matrix W, the fixed portion of cost should be isolated. For example, if during the base year for which the transaction matrix is established, the amount of personnel services funds that went to the School Center is 4,500,000 pesos. Then, by examination of the unit's book of accounts, it was determined that 350,000 pesos had been expended on activities not related to the level of forces or that this amount remains as a recurring annual expenditure independent of the level of forces. The entry in the transaction

0.225	1.960	20.588	3.8760	1.633	33.603	80.445	7.533
1.000	1.123	12.254	9.9668	0.403	20.242	63.737	1.318

TABLE 5
B MATRIX

4446.06	4641.84	4938.80	4012.82	4661.98	4975.19	7438.36	2222.49
19709.78	2660.56	2939.7	10318.69	1151.10	2997.10	5893.47	388.93

TABLE 6
NEW W MATRIX

matrix for the personnel services budgetary program should therefore be only 4,150,000 pesos. In projecting cost associated with a new level of forces, this fixed cost would be added to the estimate of personnel services fund requirement of the School Center derived from input-output analysis or the new value of w_{15} in the illustrative example.

C. TEST OF THE MODEL

There are two methods of testing the hypothesis of "constant' input coefficients, according to Hatanaka. In the first method, the input coefficients for different periods are computed and compared. Hatanaka called this method as the test by direct observation. The second method which he called as the test by output prediction, uses the data of input coefficients for a single period and the data of real outputs and final demands for different periods. Changes in the input coefficients are not directly examined, and instead, the reflection of these changes on the prediction of real outputs are analyzed.[38]

Christ pointed out that the rationale of the test is straightforward: if the predictions are good, input-output analysis is shown as a useful tool even though certain theoretical propositions are violated. If predictions turn out bad, input-output analysis proves to be impractical. He maintained however, that it would be possible for predictions to be bad even if input-output were a useful tool,

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[38] Hatanaka, pp. 62-82

if enough error or inconsistency were introduced in preparing the original matrix and all the final demand and total output figures for the prediction year (in an economy setting), and converting the latter to base year prices. Tests therefore are really tests of the data as well as of the input-output technique. If failure is indicated, the test does not indicate where it lies.[39]

In the tests by output prediction, a major problem is in determining the extent at which errors can be tolerated in the prediction of outputs in the model. Hatanaka pointed out that the answer to the problem depends on the specific purpose for which the model is utilized. In order for the model to achieve its objective, the errors in its prediction of the real outputs should not be greater than the errors in the prediction of real outputs by the other models. As a minimum requirement on the input-output model, this must be imposed regardless of the specific purpose of the model.[40]

The test that this writer would suggest for the cost model presented in this paper is generally a comparison of the result of the prediction of the model with the result of some other models designed for the cost estimation for which an input-output model is created. Definitely, an estimating tool is better than none at all. It is of the

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[39] Carl F. Christ, A Review of Input-Output Analysis, Input-Output Analysis: An Appraisal, pp. 167-168

[40] Hatanaka, p. 70

belief of this writer that with the present state of the art, there is no methodology that can significantly better the input-output analysis approach at support cost estimation, considering the quickness in which it can lead to fairly accurate estimates.

VI. CONCLUSION

In comparing the weaknesses of input-output analysis with the potential gains that could be derived in formulating a cost methodology based on the technique, one may agree that indeed input-output analysis can be a useful tool in planning and decision analysis. Of course, input-output analysis is not the only method that can provide the analysts, planners, and decision makers with the information they need. There are some other techniques like multiple regression analysis which can be used for cost estimation. However, input-output analysis could be of more appeal because of the straight-forward induction it seems to suggest in deriving easy answers to difficult problems. The consistency and quickness with which the model derives estimates of indirect costs especially, are features which cannot be ignored when one is concerned in finding a tool for cost estimation.

Being a deterministic model, input-output analysis cannot account for variation in its estimate. However, resource managers are mostly interested in point estimates. When they ask about estimates, they almost always expect the most likely value or best estimate. Deterministic models would fit into this role.

The constancy of input coefficients is the most controversal hypothesis of an input-output model. There is no doubt that the coefficients would not be constant. They

would change ultimately after some period of time. However, if cost estimates are confined to semestral or annual projections only, the change in the value of the coefficients may not be sufficient to cause marked errors in prediction or forecasting.

The extent of detail an input-output model can offer is limited only by the amount of preparation, availability of data, and level of aggregation imposed on analyst. Its versatility promises a wide range of application. In using the model, however, one should not be lulled into unwarranted confidence. Like all cost models, it is subject to uncertainties of one kind or another.

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